McGill University

PHYS 101

(Introduction to Mechanics for the Life Sciences)

FINAL EXAM SOLUTIONS

2006 PHYS 101 Final exam

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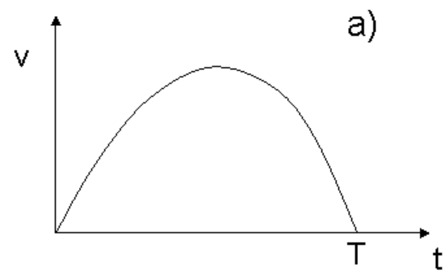
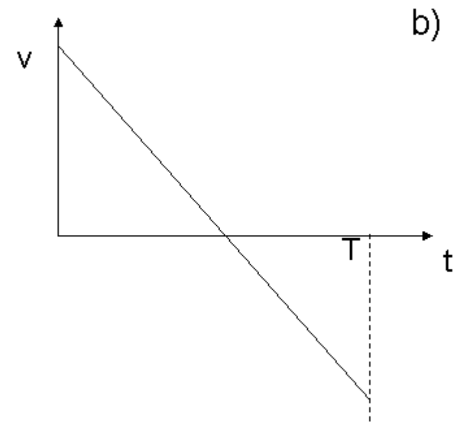
Short answer questions (answer all): you should not need to do any calculations for these questions, and should answer in **a few words, a few short phrases, or a simple sketch.** In some cases you might find it useful to quote an appropriate formula.

1) [2 pts] Tarzan is perfecting the timing of his vine-swinging, hoping to be able to impress Jane. He wonders if he has to change his timing when he has Jane on the vine with him. Assuming that the vine length is the same, what can you say about the period of the swing of the vine when he is alone, or with Jane? Assume small amplitude swings.

   Solution: The period of a pendulum is independent of mass, for small amplitude swings: Tarzan doesn't have to worry about his timing (at least, not his pendulum timing!).

2) [2 pts] For the two plots of velocity vs. time (v vs. t) shown below, which, if any, best represents the motion of a ball thrown vertically up in the air and caught some time T later?





<u>Solution</u>: b). The velocity starts as a large positive number (upwards moving), and ends as a large negative number (downwards going). The slope of a velocity vs. time graph is the acceleration, and we know that this motion is characterized by constant a (at -9.8 m/s^2). So b) correctly describes the motion.

a) describes a motion with NET displacement (the area under the curve), velocity ALWAYS positive, and acceleration ALWAYS changing.

3) [2 pts] Would you expect sound waves, or light waves, to diffract more (ie, have a larger diffraction angle) when they pass through the same opening such as a doorway? Why?

<u>Solution</u>: You would expect sounds waves to diffract more (ie, have a larger characteristic diffraction angle). That angle is $\theta = \lambda/D$ where D is the opening (doorway) size. Since $\lambda_{sound} >> \lambda_{light}$, $\theta_{sound} >> \theta_{light}$.

4) [2 pts] Two balls of different mass (but equal size) are thrown vertically upwards into the air, with the same initial velocity. They do not collide. Which ball, if any, reaches the greatest height ?

<u>Solution</u>: They reach the same height. The initial velocity is the same, and the final height is given by $d = v_o^2/2g$, independent of mass.

5) [2 pts] A physics student plays with two examples of simple harmonic motion: a mass on a spring and a simple pendulum. She finds that each of the two has a period of 1.0 sec. She's an astronaut, and on her next trip to the moon takes each along and measures the periods there (on the surface of the moon). For each system, state whether she will find the new period to be shorter than 1.0 s, longer than 1.0 s, or equal to 1.0 s (you don't need to calculate the new periods).

<u>Solution</u>: The period of the mass on the spring depends only on k (spring constant) and the mass m. Neither change on the moon, so the period is unchanged. The period of a pendulum depends on its length L and the local

acceleration due to gravity, as $T = 2\pi\sqrt{(L/g_{local})}$. On the moon, g_{local} is smaller (by a factor of about 6), so the period is longer.

6) [2 pts] You are standing on Sherbrooke Street and you hear an emergency vehicle siren at 580 Hz. You know that the 'natural' frequency of the siren (if you are motionless with respect to the vehicle) is 600 Hz. Should you jump out of the way? Why or why not?

   <u>Solution</u>: don't bother (to jump). The frequency you hear is lower, so the vehicle is moving away from you.

7) [2 pts] A rocket following a parabolic path suddenly explodes. What can you say about the resulting motion of the 'system' composed of all of the pieces?

   <u>Solution</u>: The center of mass of all the pieces will continue to follow the same parabolic path.

8) [2 pts] Using the concept of torque, explain how a screwdriver helps you to loosen a tightly attached screw.

   <u>Solution</u>: The screwdriver allows you to apply a force at a greater distance r_{driver} from the screw than its radius r_{screw}, thus allowing you to apply a much greater torque (τ=rF) to remove the screw.

9) [2 pts] A beacon in a lighthouse is supposed to produce a parallel beam of light. The beacon consists of a light bulb and a single converging lens. Where should the bulb be placed to produce the parallel beam?

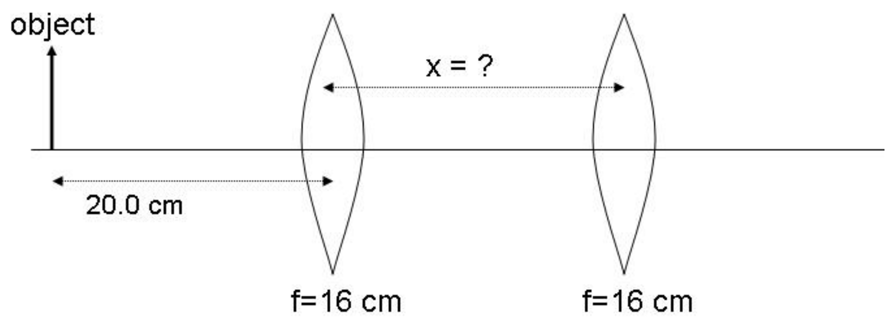
   <u>Solution</u>: The bulb should be placed at the focal distance f from the lens.

10) [2 pts] An astronaut in deep space is taking a space walk and the tether connecting him to the spaceship breaks while he is at rest (with respect to the spaceship). He is carrying only a large wrench. How can he get back to the ship?

   <u>Solution</u>: By jettisoning the wrench, in the direction <u>opposite</u> the direction to the ship. By conservation of momentum (or equivalently, by equal and opposite forces), if the wrench goes in one direction, he will go in the other.

Long problems (do five out of seven):

1) [10 pts] Two converging lenses, each of focal length f = 16.0 cm, are arranged with an object 20.0 cm to the left of one lens. The image formed by the combination of lens is upright and of the same size as the original object. Find the distance x between the lenses.



Solution: Take the lenses one at a time. For the first one:

1/do + 1/di = 1/f = 1/0.16
1/0.20 + 1/di = 1/0.16
di = 0.80 m

and the magnification is

m = -di/do = -0.80/0.20 = -4.0

This image must become the object for the second lens. We have a total magnification of +1.0, so the magnification of the second must be -0.25. Thus,

-di/do = -0.25 [second lens]
di = 0.25 do [second lens]

For the second lens, do will be x-0.80 (and the sign will be correct; if x is less than 80 cm, then do will be negative), so:

1/do + 1/di = 1/do + 1/(0.25 do) = 5/do = 1/f = 1/0.16
do = 0.80

so

x = 1.60 m

2) [10 pts] A mass of 500 grams is vibrating in simple harmonic motion according to the equation

x = 0. 60 cos(6.40 t)

where x is in meters and t is in seconds. Find:

a) the <u>amplitude</u> of the motion
b) the <u>frequency</u> of the harmonic motion
c) the <u>total energy</u> in the system

<u>Solution</u>:
a) amplitude **A = 0.60 m**
b) frequency: $\omega = 2\pi f = 6.40$ so **f = 1.02 Hz**
c) the energy is $\frac{1}{2} m v_{max}^2$ so we must find vmax.

$$v_{max} = 2\pi A/T = 2\pi f A = 6.40 \times 0.60 = 3.84 \text{ m/s}$$

So $$\mathbf{E = \tfrac{1}{2} mv^2 = 3.69\ J}$$

3) [10 pts] A small ball (of mass 750 grams) is swung at the end of a string in a horizontal circle of radius 1.5 m. Find:

a) the <u>moment of inertia</u> of the ball around the centre of the circle
b) the <u>torque</u> needed to keep the ball at constant speed (constant angular velocity) if air resistance exerts a force of 0.35 N on the ball.

You should treat the ball as a point mass and ignore the mass of the string.

<u>Solution</u>:
a) It's a point mass, so the moment of inertia is mr^2:

$$\mathbf{I = mr^2 = 1.68\ kg\ m^2}$$

b) To keep it at constant angular velocity there must be no net torque, so the applied torque must balance the torque of the air resistance.

$$\boldsymbol{\tau} \mathbf{= r\ F = 1.5\ m \times 0.35\ N = 0.525\ Nm}$$

4) [10 pts] A golf ball (mass = 39 grams) is hit off of a tee at a speed of 52 m/s. The golf club was in contact with the ball for 3.7 milliseconds (3.7×10^{-3} s). Find:

a) the <u>impulse</u> imparted to the ball
b) the <u>average force</u> exerted on the ball by the club

<u>Solution</u>:
a) The impulse is $F\Delta t = \Delta p$. The ball starts at rest, so $\Delta p = m \times v_{final}$
Thus: **impulse = 0.039 x 52 = 2.03 kg m/s**
b) The average force is then impulse/Δt

$$\mathbf{F_{avg} = 2.03/3.7 \times 10^{-3} = 548\ N}$$

5) [10 pts] Two piano strings are supposed to be vibrating at 131 Hz (this is approximately the note that is one octave below middle C on a piano). But when they are played together, you hear 3 beats every 2.0 seconds. If one of them is truly at 131 Hz, find:

a) the <u>other frequency</u> (there are two possible answers; choose one);
b) the <u>percentage by which the tension of the string must be adjusted</u> to bring it into tune at 131 Hz (indicate whether the tension should be increased or decreased for the answer you have chosen in a)

<u>**Solution**</u>:

a) The beat frequency is 1.5 Hz (3 beats in 2 seconds), so that's the difference between the two frequencies of the strings. Thus, the second frequency is either **132.5 Hz or 129.5 Hz**.
b) We have to adjust the string tension according to:

$$v = \sqrt{F_T/[m/L]}$$

and

$$v = \lambda f$$

so that:

$$f \sim v \sim \sqrt{F_T}$$

or: $$F_T \sim f^2$$

Thus, to get from 132.5 Hz to 131. Hz we must change tension by a factor of

$$(131/132.5)^2 = 0.977$$ **(a 2.3% decrease)**

and to change 129.5 Hz to 131, we must change the tension by

$$(131/129.5)^2 = 1.0233$$ **(a 2.3% increase)**

5) [10 pts] Light of wavelength 540 nm (green) <u>in air</u> falls on a two-slit apparatus where the two slits are 7.50×10^{-2} mm apart. However, the two-slit apparatus is immersed in water (n=1.33). How <u>far apart</u> are the bright fringes on the screen that is 50 cm away from the slits?

<u>**Solution**</u>:

This is a regular two-slit interference question, but the wavelength will change when the apparatus is immersed in water, with λ becoming λ/n.
The standard interference equation, giving the angles to the bright fringes, is:

$$d \sin(\theta) = m \lambda \qquad m=0,1,2,\ldots$$

The distance between the fringes is given by comparing m=0 (where θ=0) to m=1, where

$$\sin(\theta) = \lambda/(d\, n)$$
$$\sin(\theta) = 540 \times 10^{-9}/(7.5 \times 10^{-5} \times 1.33) = 5.41 \times 10^{-3}$$

This is small, so we can use the small-angle approximation to calculate θ in radians, then we get the fringe spacing by:

$$x = \theta L$$

where L=0.50 m is the distance from the slits to the screen.

$$x = 5.41 \times 10^{-3} \times 0.50 = 2.70 \times 10^{-3} \text{ m} = \mathbf{2.70\ mm}$$

6) [10 pts] A train is traveling at a constant speed around a curve of radius 275 m. A lamp suspended in the train swings out to an angle of 19^o from the vertical. What is the <u>speed</u> of the train?

<u>**Solution**</u>:

The free body diagram has mg downwards, and FT (the tension in the chain holding the lamp) at an angle $\theta=19^o$ from the vertical (and pointing towards the center of the circle.

The vertical components, mg and $FT\cos(19^o)$ must balance (the lamp doesn't have any acceleration in that direction, so we have:

$$mg = F_T \cos(19^o)$$
$$F_T = mg/\cos(19^o)$$

The horizontal component of the tension in the chain is unbalanced – because there's a net force (which must be the centripetal force). So we have:

$$mv^2/r = F_T\sin(19^o) = mg\ \sin(19^o)/\cos(19^o) = mg\ \tan(19^o)$$

and thus

$$v^2 = gr\ \tan(19^o) = 9.8 \times 275 \times \tan(19^o) = 928$$
$$\mathbf{v = 30.5\ m/s}$$